keep it simple science

KEEP IT SIMPLE SCIENCE
Physics Module 5

Advanced Mechanics
WORKSHEETS

Worksheet 1 Projectiles
Practice Problems

Student Name....................................

1. For each of the following projectiles, resolve the initial launch velocity into horizontal and vertical components.

a) A rugby ball kicked upwards at an angle of 60º, with velocity 20.5ms^{-1}.

b) A bullet fired horizontally at 250ms^{-1}.

c) A baseball thrown at 15.0ms^{-1}, and an up angle of 25º.

d) An artillery shell fired at 350ms^{-1}, upwards at 70º.

e) An arrow released from the bow at 40.0ms^{-1}, at 45º up.

2. For the arrow in Q1(e), find
a) the time to reach the highest point of its arc.

b) the maximum height reached.

c) its range (on level ground).

3. The bullet in Q1(b), was fired from a height of 2.00m, across a level field. Calculate:
a) how long it takes to hit the ground.

b) how far from the gun it lands.

c) At the same instant that the bullet left the barrel, the empty bullet cartridge dropped (from rest) from the breech of the gun, 2.00m above the ground. How long does it take to hit the ground? Comment on this result, in light of the answer to (a).

4. For the artillery shell in Q1(d), calculate:
a) the time to reach the highest point of its arc.

b) the maximum height reached.

c) its range (on level ground).

5. The rugby ball in Q1(a) was at ground level when kicked.
a) Find its exact position 2.50s after being kicked.

b) What is its instantaneous velocity at this same time?

keep it simple science

Worksheet 2 More About Projectiles

Fill in the blanks

Student Name.....................................

A projectile is any object which is launched, and then moves a)................................. The path of a projectile is called its b)..............................., and is a curve. Mathematically, the curve is a c).................................

To analyse projectile motion it is essential to treat the motion as 2 separate motions; d)................................. and ....

If the launch velocity and the e)........................... of launch are known, you should always start by f)..................................... the initial velocity into horizontal and vertical g)...............................

The horizontal motion is always h)....................... .... and the vertical is constant i).................................. due to j)............................

The usual strategy is to find the k)...................... of flight, by using the fact that at the top of the projectile's arc its vertical velocity is l)..............

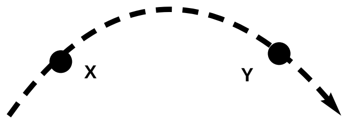
Once this is known, it becomes possible to calculate the maximum m).............................. attained, and the n)........................... (total horizontal displacement.). The projectile's position and velocity at any instant can be found by combining the o)............................ and .... vectors.

Maximum range of any projectile occurs when the angle of launch is p)................... degrees upwards.

Worksheet 3 Even More Projectiles

Student Name.....................................

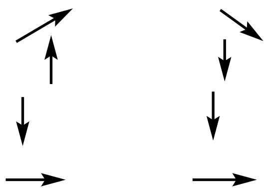
Multiple Choice



1. The diagram shows the trajectory of a projectile, and 2 points X & Y.
Which pair of vectors below correctly identifies the total acceleration vector of the projectile at points X and Y?

Point X Point Y

A.
B.
C.
D.

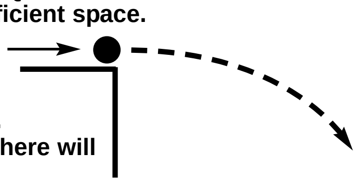


2.
To analyse projectile motion mathematically, usually the first thing to do is to:
A. find the time of flight.
B. calculate the range.
C. calculate the maximum height reached.
D. resolve the initial velocity into vertical & horizontal components.

3.
Ignoring air-resistance, the maximum range for any projectile (for the same launch velocity) will occur when:
A. it is launched horizontally.
B. it is launched at 45° upwards.
C. it is launched to achieve a greater height.
D. its vertical acceleration is increased.

Longer Response Questions

Answer on reverse if insufficient space.



4.
A ball was rolled along a horizontal table at $5.45 ms^{-1}$.
If the table is 1.20m high, where will the ball hit the ground?

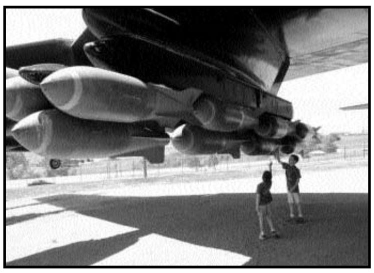
5.
An arrow was released from the bow at an upward angle of 60° and an initial velocity of $42.0 ms^{-1}$. It hits its target at the same horizontal level from which it was released.
a) Find the time of flight.

b) Find the maximum height reached.

c) Calculate the distance from bow to target.

6.

Photo: Arian Kulp



These military bombs are designed to be dropped from the aircraft at an altitude of 15,000m when the plane is in level flight at a velocity of $300 ms^{-1}$.

a) Ignoring air-resistance, how far in front of the target must the bombs be released?

b) How fast will they be going (magnitude only) when they hit the ground?

keep it simple science

Worksheet 4 Circular Motion Practice Problems

Student Name....................................

1.
A 750g ball is swung in a circle on a string 1.75m long. It completes 10 revolutions in 6.5 s.
a) What is the period?

b) Find its orbital speed.

c) What is its centripetal acceleration?

d) Centripetal force?

2.
A 3,000kg aircraft is flying at 300 km/hr in level flight, and begins a circular turn with radius 500m.

a) What centripetal force is needed to effect this turn? (Hint: first convert velocity to m/s)

b) How long will it take to complete a 180^o turn?

3.
a) The maximum "grip" force of each tyre on a 1,000kg car is 4,500N. What is the tightest turn (in terms of radius of curve) the car can negotiate at 90 km/hr? (Hint: velocity units?)

b) The same car comes to a curve with <u>double</u> this radius, (ie a much gentler curve) but it is travelling at <u>double</u> the speed. Can it make it?

4.
a) What is the angular velocity of the ball in Q1?

b) What is the angular velocity of the plane in Q2?

c) What is the angular velocity of the car in Q3(a)?

5.
A wheel is rotating at 1,000 RPM.
a) What is the period of the rotation?

b) What is its angular velocity?

c) What is its orbital speed, if the radius is 0.8m?

d) What is the centripetal acceleration?

6.
A rotating "ferris wheel" amusement park ride has a radius of 30m and rotates once each 45s.
a) What is its angular velocity?

c) What is the orbital speed?

7.
A boat on a lake is tethered by a rope to a stong post. The boat is able to drive around the post in a circle by always pointing at a tangent to the circle. The boat's orbital speed through the water is constant, but the rope keeps shortening as it winds around the post.
a) Show mathematically what will happen to the angular velocity as the rope shortens.

a) Show mathematically what will happen to the tension in the rope as it shortens.

keep it simple science

Worksheet 5 Torque Talk
Questions

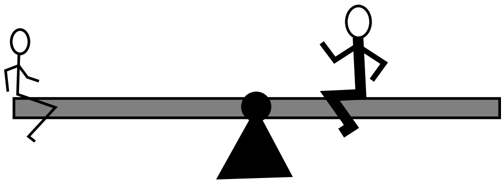
Student Name....................................

1.
a) Explain why using a hand tool such as this wrench (spanner) is all about applying a torque.

b) If you are having trouble undoing a rusty bolt, one “trick” is to use a spanner with a longer handle. Explain the Physics.

2.
Calculate the torque in each case:
a) A force of 100N acts at a point 40cm from a pivot point at a 20^o angle to the lever arm.

b) The same force is applied at the same point, but at an angle of 90^o.



3.
How can this see-saw be perfectly balanced by a heavy adult and a small child?
Explain this in terms of torque.

4.
This playground toy spins in a circle, but is its Physics the same as (say) an object being swung around on a string? Discuss similarities & differences.

Worksheet 6 Newton’s Gravity
Practice Problems

Student Name.....................................

1. Fred (75kg) and girlfriend Sue (60kg) are very much attracted to each other, but is it love or just gravity?

Calculate the gravitational force attracting them when they are 0.5m apart.

2. What is the gravitational force of attraction between 2 small asteroids with masses of 6.75×10^8kg and 2.48×10^9kg separated by 425m?

3. The mass of the Moon is 6.02×10^{22}kg. A comet with mass 5.67×10^{10}kg is attracted to the Moon by a force of 6.88×10^{10}N. How far apart are the 2 bodies?

4.
What is the gravitational force between the Earth and the Moon? (Distance Earth-Moon = 3.84×10^8m)

5. <u>Research</u>: The first person to measure a value for the constant “G”, was a strange little Englishman called Cavendish. How did he do it? (& why “strange”?)

keep it simple science

Worksheet 7 Gravitational Field Strength
Calculation Exercise

Student Name....................................

Use $g = GM/r^2$ to complete this table for selected members of our Solar System.

This Column is for Worksheet 9

Planet	Mass (kg)	Radius (m)	Surface Gravity "g" ($N.kg^{-1}$ or ms^{-2})	"g" as multiple of Earth's	Escape Velocity (ms^{-1})	(km/s)
Earth	6.0×10^{24}	6.371×10^{6}	9.86 (calculated from these data)	1.0	1.12×10^{4}	11.2
Mercury	3.3×10^{23}	2.44×10^{6}	(a)	(b)	p)	q)
Venus	4.9×10^{24}	(c)	(d)	0.904	r)	s)
Saturn	(e)	5.8×10^{7}	10.44	(f)	t)	u)
Pluto	1.3×10^{22}	(g)	0.62	(h)	v)	w)

Worksheet 8 Mass & Weight
Practice Problems

Student Name....................................

1.
A small space probe has a mass of 575kg.

a) What is its <u>mass</u>
i) in orbit?

ii) on the Moon?

iii) on Jupiter?

b) What is its <u>weight</u>
i) on Earth?

ii) on the Moon?

iii) on Jupiter?

2.
If a martian weighs 250N when at home, what will he/she/it weigh:
a) on Earth? (hint: firstly find the mass)

b) on Neptune?

c) on the Moon?

3.
A rock sample, weight 83.0N, was collected by a space probe from the planet Neptune.
a) What is its mass?

b) What will it weigh on Earth?

c) On which planet would it weigh 206N?

keep it simple science

Worksheet 10 Satellites & Orbits

Fill in the blanks

Student Name.......................................

If a projectile is travelling horizontally at the correct a)..................................., then its down-curving trajectory will match the b)............................. of the Earth. The projectile will continue to "fall down" but never reach the surface... it is now a c).......................... which is d)................................ around the Earth.
To place a satellite in orbit, it must be e)................................... up to orbital speeds.

During upward acceleration, an astronaut will experience "f)..........................." which feel like an increase in g)......................... and can be life-threatening if too high.

The only feasible technology (so far) for achieving the necessary h)................................., while keeping the i)................................. reasonably low, is the use of j).........................

One of the important steps in the history of rocketry was achieved by Robert Goddard, who built and tested the first k)............................-fuelled rocket.
Rockets are always launched towards the l)...................... to take advantage of the Earth's m)................................

Rocket propulsion is a consequence of Newton's n)........... Law. During the launch, momentum is o)............................. The backward momentum gained by the exhaust gases is matched by the p).......................... momentum gained by the q)............................. However, the mass of the rocket r)....................... rapidly as is burns huge amounts of fuel. This means that even with constant thrust, the acceleration rate s)................................, and the astronauts feel increasing t).............................. unless the engines are throttled back.

There are basically 2 different types of orbit for a satellite: u)....................................... orbits are when the satellite is v).......................... km from Earth and travelling very w).............................. This is ideal for satellites used for x)................................. and .... The other type of orbit is called y)..................................... For this the satellite is positioned above the z)........................ so its aa).......................... is exactly 24 hrs. This means it has the same ab).............................. velocity as the Earth, and seems to stay in the ac)................... .... in the sky. This is ideal for ad)................................ satellites.

Worksheet 11 Satellite Orbits

Student Name.......................................

Practice Problems

Use the equations of Circular Motion.

1.
A satellite orbiting 1,000km above the Earth's surface has a period of 1.74 hours. (Radius of Earth=6.371×10^6m)
a) Find its orbital velocity, using $v=2\pi\ r / T$

b) If the satellite has a mass of 600kg, find the centripetal force holding it in orbit.

2.
A 1,500kg satellite is in Earth orbit travelling at a velocity of 6.13 km/s ($6.13 \times 10^3 ms^{-1}$). The Centripetal force acting on it is 5.32×10^3N.

a) What is the radius of its orbit?

2. (cont)
b) What is its altitude above the earth's surface?

c) What is the period of its orbit?

3.
A satellite is being held in Earth orbit by a centripetal force of 2,195N. The orbit is 350km above the Earth & the satellite's period is 1.52 hrs.
a) Find the orbital velocity.

b) What is the satellite's mass?

keep it simple science

Worksheet 12 Using Kepler's Law
Calculation Exercise

Student Name....................................

1.
The Earth takes 1 year to complete an orbit around the Sun, with an orbital radius about 150 million km.
a) Using these arbitrary units (years, millions km) calculate a value for radius cubed, divided by period squared. (r^3 / T^2)

b) What is the significance of this ratio value for all the planets of the Solar System?

2.
Use this value to find the orbital radius of Jupiter, given that it takes 11.8 Earth years to complete an orbit around the Sun.

3.
Find the orbital period of Mars, given that its radius of orbit is about 228 million km.

4.
Mercury orbits only 58 million km from the Sun. How long is a "Mercurian year", in Earth days?

5.
The minor planet Pluto takes 248 Earth years to complete an orbit. What is its (average) orbital radius? (its orbit is highly elliptical)

6.
a) Research: Find out what is meant by an "Astronomical Unit" (AU).

b) What would be the value of r^3 / T^2 for Earth if we used units of AU and years?

Worksheet 13 Energy of a Satellite
Practice Problems

Student Name....................................

1.
Arange these values in order of increasing size.

-2×10^6, -8×10^4, -9×10^{10}, -5×10^5, -6×10^6

2.
a) Calculate the total energy of a 5,000kg satellite in an Earth orbit with radius 5×10^7 m.

b) The same satellite is brought down to a lower orbit of radius 2×10^7 m. Calculate the new total energy and hence the energy change.

c) Explain which form(s) of energy have been lost or gained.

3.
a) For the satellite in its higher orbit in Q2, use $v^2 = GM/r$ to calculate its orbital velocity in this orbit.

b) Use $E_k = 1/2\ mv^2$ to find its kinetic energy.

c) Use $E_k = GMm/2r$ to find its kinetic energy.

4.
About 65 million years ago, life on Earth was severely disrupted by the collision of an asteroid or comet about 10km in diameter. Its mass can be estimated at 3×10^{15} kg. Calculate its total energy as it hit the surface of the Earth.

keep it simple science

Worksheet 14 Module Summary
Guided Notes.
(Make your own summary)

Student Name.....................................

Projectiles
1.
Summarise the main characteristics of the Physics of projectile motion.

2.
Describe how you would go about finding the range of a projectile, given its launch velocity & angle.

Circular Motion
3.
Derive, from first principles, an expression for the orbital speed of an object in circular motion.

4.
Differentiate between
a) "centrifugal force" and "centripetal force".

b) "orbital velocity" and "angular velocity".

5.
How much "work" is done by a centripetal force?

6.
What is "torque" a measure of?

Orbits & Satellites
7.
What is the effect on the gravitational force between 2 masses if:
a) one mass is doubled?

b) distance between them is increased by 4 times?

c) distance is <u>decreased</u> to 1/10?

8.
a) Explain the notion of a gravitational orbit as outlined by Newton.

b) What is meant by "escape velocity"?

9.
Explain why we use rockets to launch a spacecraft, rather than any other method.

10.
Relate the different satellite orbits to their uses.

11.
a) Outline Kepler's "Law of Periods".

b) Write the maths of Newton's proof of this Law.

12.
a) Define "Grav. Potential Energy".

b) What is the consequence of this definition?

keep it simple science ®

Answer Section

Worksheet 1

1.

$u_y = u.\sin\theta$ $\quad$ $u_x = U.\cos\theta$

a) $= 20.5\text{x}\sin 60$ $\quad$ $= 20.5\text{x}\cos 60$
$= 17.8\text{ms}^{-1}$. $\quad$ $= 10.3\text{ms}^{-1}$.

b) vertical = zero $\quad$ horizontal = 250ms^{-1}.

c) $u_y = 15.0\text{x}\sin 25$ $\quad$ $u_x = 15.0\text{x}\cos 25$
$= 6.34\text{ms}^{-1}$. $\quad$ $= 13.6\text{ms}^{-1}$.

d) $350\text{x}\sin 70$ $\quad$ $350\text{x}\cos 70$
$= 329\text{ms}^{-1}$. $\quad$ $= 120\text{ms}^{-1}$.

e) $40.0\text{x}\sin 45$ $\quad$ $40.0\text{x}\cos 45$
$= 28.3\text{ms}^{-1}$. $\quad$ $= 28.3\text{ms}^{-1}$.

2.

a) At highest point, $v_y=0$, and $v_y = u_y + g.t$
$0 = 28.3 + (-9.81\text{x } t)$
$t = -28.3/-9.81$
$= 2.88\text{s}.$

b) $S_y = u_y.t + \frac{1}{2}.g.t^2$
$= 28.3\text{x}2.88 + (0.5\text{x } (-9.81) \text{ x } 2.88^2)$
$= 81.5 + (-40.7) = 40.8\text{m}.$

c) $S_x = v_x.t = 28.3 \text{ x } (2.88\text{x}2)$
(twice the time to reach max.ht.)
$= 163\text{m}.$

3.

a) It is fired from max height,
so $S_y = -2.00$ (down, so -ve)
$S_y = u_y.t + \frac{1}{2}.g.t^2$
$-2.00 = 0\text{x}t +(0.5\text{x}(-9.81)\text{x } t^2)$
$-2.00 = 0 - 4.905 \text{ x } t^2$
$t^2 = -2.00/-4.905$
$t = 0.639\text{s}.$

b) $S_x = v_x.t = 250\text{x}0.639 = 160\text{m}.$

c) see working for (a).
Empty cartridge takes 0.639s to hit the ground. It falls down at exactly the same rate as the bullet. The difference is where each lands horizontally.

4.

a) At highest point, $v_y=0$, and $v_y = u_y + g.t$
$0 = 329 + (-9.81)\text{x } t$
$t = -329/-9.81$
$= 33.5\text{s}.$

b) $S_y = u_y.t + \frac{1}{2}.g.t^2$
$= 329\text{x}33.5 + (0.5\text{x}(-9.81)\text{x}33.5^2)$
$= 11{,}022 - 5{,}505$
$= 5{,}517 = 5.52\text{x}10^3\text{m}.$

c) $S_x = v_x.t = 120\text{x}(33.5\text{x}2)$
(twice the time to reach max.ht.)
$= 8{,}040 = 8.04\text{x}10^3\text{m}.$

5.

a) <u>Vertical displacement</u>
$S_y = u_y.t + \frac{1}{2}.g.t^2$
$= 17.8\text{x}2.50 + (0.5\text{x}(-9.81)\text{x}2.50^2)$
$= 44.5 + (-30.65)$
$=13.4\text{m}$ (+ve, therefore up)

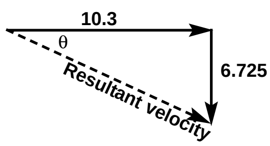
<u>Horizontal Displ.</u>
$S_x = v_x.t$
$= 10.3 \text{ x } 2.50$
$= 25.8\text{m}$

Ball is 25.8 metres down-field and 13.4 m high.

b) <u>Vertical velocity</u>
$v_y = u_y + g.t$
$= 17.8 + (-9.81)\text{x}2.50$
$= -6.725\text{ms}^{-1}$ (downwards)

<u>Horizontal velocity</u>
$v_x = u_x = 10.3 \text{ ms}^{-1}$

$v^2 = v_y^2 + v_x^2 = 10.3^2 + 6.725^2$



$\therefore v = \sqrt{151.32} = 12.3\text{ms}^{-1}.$
$\tan\theta = 6.725/10.3,$
$\therefore \theta \cong 33^o$ below horizontal

Worksheet 2

a) only under gravity
b) trajectory
c) parabola
d) horizontal & vertical
e) angle
f) resolving
g) components
h) constant velocity
i) acceleration
j) gravity
k) time
l) zero
m) height
n) range
o) horizontal & vertical
p) 45

Worksheet 3

1. C $\quad$ 2. D $\quad$ 3. B

4.

$u_y=0$, $u_x=5.45\text{ms}^{-1}$, $S_y = -1.20\text{m}$ (down (-ve))
Time of flight: $S_y = u_y.t + 0.5.g.t^2$
$-1.20 = 0\text{x}t + (0.5\text{x}(-9.81)\text{x}t^2)$
$t = \text{sq.root}(-1.20/-4.905)$
$= 0.495\text{s}.$
Horizontal distance: $S_x = u_x.t = 5.45\text{x}0.495 = 2.95\text{m}.$
The ball lands 2.95m from the base of the table.

5.

$u_y = u.\sin\theta$ $\quad$ $u_x = u.\cos\theta$
$= 42.0\text{x}\sin 60$ $\quad$ $= 42.0\text{x}\cos 60$
$= 36.4\text{ms}^{-1}$ $\quad$ $= 21.0\text{ms}^{-1}$.

a) At max.height, $v_y = 0$,
and $v_y = u_y + g.t$
$0 = 36.4 \text{ x } (-9.81)\text{x } t$
$t = -36.4/-9.81$
$= 3.71\text{s}$ (to highest point)
Time of flight = 3.71x2 = 7.42s.

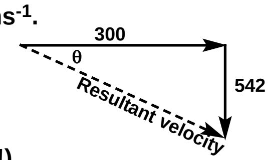
b) $S_y = u_y.t + 0.5.g.t^2$ (use time to highest point)
$= 36.4\text{x}3.71 + (0.5\text{x}(-9.81)\text{x}3.71^2)$
$= 135 + (-67.5) = 67.5\text{m}.$

c) Range: $S_x = u_x.t = 21.0\text{x}7.42$ (Time for entire flight)
$= 156\text{m}.$

6.

a) $u_y=0$, $u_x=300\text{ms}^{-1}$, $S_y = -15{,}000\text{m}$ (down (-ve))
Time of flight: $S_y = u_y.t + 0.5.g.t^2$
$-15{,}000 = 0\text{x}t + (0.5\text{x}(-9.81)\text{x}t^2)$
$t = \sqrt{(-15{,}000/ -4.905)}$
$= 55.3\text{s}.$
Horizontal distance: $S_x = u_x.t = 300\text{x}55.3 = 16{,}590\text{m}$
$= 1.66\text{x}10^4\text{m}.$
Bombs must be released over 16km before the target.

b) $V_y = u_y + g.t$ $\quad$ $u_x=300\text{ms}^{-1}$.
$= 0 + (-9.81)\text{x}55.3$
$= 542\text{ms}^{-1}.$



$v^2 = v_y^2 + v_x^2 = 542^2 + 300^2$
$\therefore v = \sqrt{383{,}764} = 619\text{ms}^{-1}.$
(almost twice the speed of sound!)

Don't forget that we are assuming no air resistance.
In the real world, these answers would be quite different.

keep it simple science

Answer Section

Worksheet 4

1.
a) 6.5 / 10 = 0.65s = T
b) $v = 2\pi r / T$ = 2x3.142x1.75 / 0.65 = 16.9 ms^{-1}.
c) $a_c = v^2 / r$ = 16.9^2 / 1.75 = 164 ms^{-2}.
d) $F_c = m.a_c$ = 0.75x164 = 123 N.

2.
v = 300/3.6 = 83.3ms^{-1}.
a) $F = mv^2/r$ = 3,000x83.3^2 / 500 = 4.16x10^4N.

(41,600 N. That's why planes have strong wings!)
b) $v = 2\pi r / T$, so $T = 2\pi r / v$
=2x3.142x500 / 83.3 = 37.7s
A 180 turn will take half of that = 18.9s.

3.
a) v = 90/3.6 = 25ms^{-1}.
Total grip from 4 tyres = 4,500x4 = 18,000N.
$F_c=mv^2/r$, so $r=mv^2/F$ = 1,000x25^2 / 18,000 = 34.72...
= 35m.
b) r = 70m, v = 50ms^{-1}.
Centripetal force needed: $F=mv^2/R$ = 1,000x50^2/70
= 35,714N
Since the maximum grip of the tyres is only 18,000N, the tyres cannot provide the force needed to turn this corner... car will "spin out".

4.
a) $v = \omega r$ so $\omega = v / r$ = 16.9 / 1.75 = 9.66 rad.s^{-1}.
b) $v = \omega r$ so $\omega = v / r$ = 83.3 / 500 = 0.167 rad.s^{-1}.
c) $v = \omega r$ so $\omega = v / r$ = 25 / 35 = 0.714 rad.s^{-1}.

5.
a) 1,000 RPM = 1000 /60 revs/sec = 16.7 revs/sec. means that T = 0.06s.
b) $\omega = 2\pi / T$ = 2x3.142 / 0.06 = 105 rad.s^{-1}.
c) $v = \omega r$ = 105 x 0.8 = 83.8 ms^{-1}.
d) $a_c = \omega^2 r$ = 105^2 x 0.8 = 8,820 ms^{-2}.

6.
a) $\omega = 2\pi / T$ = 2x3.142 / 45 = 0.14 rad.s^{-1}.
b) $v = \omega r$ = 0.14 x 30 = 4.19 ms^{-1}.

7.
a) $v = \omega r$ so $\omega = v / r$
If v is constant, but r decreases, then ω must increase.

b) The tension in the rope is equal to centripetal force. $F_c = m \omega^2 r$

Assume mass is constant. As the rope shortens, r decreases, but ω increases. Since F is proportional to r and the <u>square of ω</u>, the force must increase.

Worksheet 5

1.
a) To use the spanner you apply force at some distance from the pivot point at the nut or bolt. This creates the torque to make it turn in a circle.
b) A longer handle allows the force to be applied at a larger distance from the pivot, which increases the torque. (for the same force)

2.
a) $\tau = r.F.\sin\theta$ = 0.4x100x sin20 = 13.7 Nm
b) $\tau = r.F.\sin\theta$ = 0.4x100x sin90 = 40 Nm

3.
The see-saw will balance if the opposing "turning moments" are equal, but in opposite directions. The heavier adult must sit closer to the pivot until
$r_1.F_1 = r_2.F_2$

4.
<u>Similarity</u>: both motions are circular and can be described by an angular or orbital velocity.
<u>Difference</u>: the play equipment rotates because of force applied tangentially at its rim. An object on a string is accelerated into a curve by a centripetal force pulling it towards the centre of rotation.

Worksheet 6

1.
$F_G = GMm/r^2$ = 6.67x10^{-11}x75x60/0.5^2
= 1.20x10^{-6}N. (about 1 millionth of a newton)

2.
$F_G = GMm/r^2$ = 6.67x10^{-11}x6.75x10^8x2.48x10^9/425
= 2.63x10^5N.

3.
$d = \sqrt{GMm/F}$
$= \sqrt{6.67\text{x}10^{-11}\text{x}6.02\text{x}10^{22}\text{x}5.67\text{x}10^{10}/6.88\text{x}10^{10}}$
= 1.82x10^6m.
(Since this equals 1,820km, and the radius of the Moon is 1,738km, then the comet is just 82km from the surface... DEEP IMPACT about to happen!)

4.
$F_G = GMm/r^2$
= 6.67x10^{-11}x6.0x10^{24}x6.02x10^{22} / (3.84x10^8)^2
= 1.63 x 10^{20} N.

5.
Hopefully, you found out some stuff about Henry Cavendish. Note that he actually measured the density (and from that the mass) of Earth. He could have determined "G", but no-one did the calculation for about 100 years. His measurements were amazingly accurate (for 1898). His value for "G", if he'd calculated it, were out by only 1%.

"Strange"? He was painfully shy, possibly due to autistism or Asperger's syndrome. He could not even speak to women & never married. Undoubtably one of the most brilliant scientists of all time. Also incredibly rich!

keep it simple science

Answer Section

Worksheet 7

a) 3.70 b) 0.38
c) 6.06×10^6 d) 8.91
e) 5.27×10^{26} f) 1.06
g) 1.18×10^6 h) 0.063

Worksheet 8

1.
a) i) 575kg ii) 575kg iii) 575kg.
b) i) F=mg = 575x9.81 = 5,641 = 5.64×10^3N.
ii) F=mg = 575x1.6 = 920 = 9.2×10^2N.
iii) F=mg = 575x25.8 = 14,835 = 1.48×10^4N.

2.
a) On Mars; F=mg, so m=F/g = 250/2.8 = 65.8kg
On Earth; F=mg = 65.8x9.81 = 645 = 6.5×10^2N.
b) On Neptune; F=mg = 65.8x10.4= 684 = 6.8×10^2N.
c) On Moon; F=mg = 65.8x1.6 = 105 = 1.1×10^2N.

3.
a)
On Neptune; F=83.0 =mg, so m= 83.0/10.4 = 7.98kg.
b) On Earth; F=mg = 7.98x9.81 = 78.3N.
c) F=206=mg, so g=206/7.98 = 25.8ms^{-2}.
matches Jupiter

Worksheet 9

p) 4.25×10^3	q) 4.25
r) 1.04×10^4	s) 10.4
t) 3.48×10^4	u) 34.8
v) 1.21×10^3	w) 1.21

Worksheet 10

a) velocity	b) curvature
c) satellite	d) in orbit
e) accelerated	f) g-forces
g) weight	h) velocity
i) g-forces	j) rockets
k) liquid	l) east
m) rotation	n) 3rd
o) conserved	p) forward
q) rocket	r) decreases
s) increases	t) g-forces
u) low-Earth	v) 200-1,000
w) quickly/fast	x) photos & surveys
y) geo-stationary	z) equator
aa) period	ab) angular
ac) same position	ad) communication

Worksheet 11

1.
a) T=1.74 hours = 1.74x60x60= 6,264s
r= 1,000 km (=10^6m) + 6.37×10^6 = 7.37×10^6m
v = $2\pi r/T$
= $2 \times \pi \times 7.37 \times 10^6/6{,}264$
= 7,393 = 7.39×10^3ms^{-1}.
b) $F_c = mv^2/r = 600 \times (7.39 \times 10^3)^2/7.37 \times 10^6$
= 4.45×10^3N.

2.
a) $F_c = mv^2/r$, so $r = mv^2/F$
= $1{,}500 \times (6.13 \times 10^3)^2/5.32 \times 10^3$
r = 1.06×10^7m.
b) Altitude = $1.06 \times 10^7 - 6.37 \times 10^6 = 4.23 \times 10^6$m
(4,230km)
c) v = $2\pi r$ / T, so T = $2\pi r/v$
= $2 \times \pi \times 1.06 \times 10^7/6.13 \times 10^3$
= 1.09×10^4s. (3.02 hours)

3.
R = 350km + 6.37×10^6m = 6.72×10^6m
T= 1.52 hrs = 1.52x60x60 = 5.47×10^3s.
a) v = $2\pi r$ / T = $2 \times \pi \times 6.72 \times 10^6 / 5.47 \times 10^3$
= 7.72×10^3ms^{-1}.
b) $F_c = mv^2/r$, so $m = F.r/v^2$
= $2{,}195 \times 6.72 \times 10^6 / (7.72 \times 10^3)^2$
= 247kg.

Worksheet 12

1.
a) If r=150 and T=1, then $r^3 / T^2 = 150^3 / 1^2$
= 3.38×10^6
b) According to Kepler's Law of Periods, all objects in orbit around the Sun will have the same value for r^3 / T^2.

2.
$r^3 / T^2 = 3.38 \times 10^6$
$r^3 = 3.38 \times 10^6 \times 11.8^2$
r = 777 million km

3.
$r^3 / T^2 = 3.38 \times 10^6$
$T^2 = r^3 / 3.38 \times 10^6$
$T = \sqrt{228^3 / 3.38 \times 10^6}$ = 1.87 years

4.
$T^2 = r^3 / 3.38 \times 10^6$
$T = \sqrt{58^3 / 3.38 \times 10^6}$ = 0.24 years= 88 days

5.
$r^3 / T^2 = 3.38 \times 10^6$
$r^3 = 3.38 \times 10^6 \times 248^2$
r = 5,920 million km

6.
a) The AU is the average radius of the Earth's orbit, = 150 million km.
b) Using AU and years, $r^3 / T^2 = 1^3 / 1^2 = 1$

keep it simple science

Answer Section

Worksheet 13

1.
-9×10^{10} -6×10^{6} -2×10^{6} -5×10^{5} -8×10^{4}

2.
a) $E_k + U = -GMm / 2r$
$= -(6.67\times10^{-11}\times6.0\times10^{24}\times5000 / 2\times5\times10^{7}$
$= -2.0 \times 10^{10}$ J

b) $E_k + U = -GMm / 2r$
$= -(6.67\times10^{-11}\times6.0\times10^{24}\times5000 / 2\times2\times10^{7})$
$= -5.0 \times 10^{10}$ J

Energy change $= -5.0\times10^{10} -(-2.0\times10^{10}) = -3.0\times10^{10}$ J

c) Moving to a lower orbit, it has gained some Ek (faster), but lost GPE (lower). Overall it has lost 30,000 MJ of energy.

3.
a) $v^2 = GM/r = 6.67\times10^{-11}\times6.0\times10^{24} / 5\times10^{7}$
$v = 2.83 \times10^{3}$ ms^{-1}.

b) $E_k = 1/2\ mv^2 = 0.5\times5000\times(2.83\times10^{3})^2$
$= 2.0\times10^{10}$ J

c) $E_k = GMm/2r = 6.67\times10^{-11}\times6.0\times10^{24}\times5000 / 2\times5\times10^{7}$
$= 2.0\times10^{10}$ J

They agree. Gotta love it when things work!

4.
$E_k + U = -GMm / 2r$
$= -(6.67\times10^{-11}\times6\times10^{24}\times3\times10^{15} / 2\times6.371\times10^{6})$
$= -9.42 \times 10^{22}$ J   THAT'S BIG

You may argue that the Maths does not apply since this object was not in orbit. Using the KISS Principle, we argue that it <u>was</u> an orbit, but that it went a bit wrong.

Worksheet 14

1.
A projectile is a moving object which is acted upon by only 1 force... gravity. Its vertical motion is constant acceleration (at g), while horizontal motion is constant velocity. Projectiles follow a parabolic path and achieve max. range when launched at 45^o.

2.
- resolve the launch velocity into horizontal & vertical components.
- use the vertical motion to find time of flight.
- use horizontal motion to find displacement in that time.

3.
For a circle of radius r, the circumference is 2π r.
Time taken for one revolution is "T".
Speed = distance / time, so the speed during one revolution is $v = 2\pi\ r / T$.

4.
a) Centripetal force is the force which pulls a moving object into circular motion. It act towards the centre of the circle.
"Centrifugal force" is a "pseudo-force" which <u>seems</u> to push things in circular motion towards the outside of the curve. However, this is only a perception of the observer who is in circular motion. When analysed from a non-accelerating "frame of reference" this force does not exist.

Worksheet 14

4.
b) Orbital speed or velocity is the rate of movement of an object in circular motion, measured in ms^{-1}, or other distance/time units.
Angular velocity is the rate of change of position in the orbit as seen from the centre of the circle, measured as angular displacement / time.

5.
None at all, because centripetal force always acts at right angles to the displacement vector which is tangential.

6.
Torque is a measure of the "turning effect" of a force applied which causes an object to rotate. It may result in circular motion, but is not the result of centripetal force acting on a moving body.

7.
a) doubles the force.
b) decreases the force to 1/16.
c) increases the force 100 times.

8.
a) He imagined a cannon firing horizontally at increasing velocities. An orbit will occur when the downward curve of the projectile matches the curvature of the Earth's surface. The cannon ball will continue to fall down, but can never reach the surface. (in absence of air resistance)
b) If fired fast enough, the cannon ball can escape completely from Earth's gravity. The velocity required is "escape velocity".

9.
Only rockets have the power to reach orbital speeds <u>and</u> work without oxygen from the air <u>and</u> can avoid high g-forces which could kill passengers.

10.
Low-Earth orbits are close enough for detailed photographic surveys (and other studies) which eventually can cover the entire surface of the Earth.
Geostationary orbits are much further out, but always appear to sit in the same position in the sky. This is ideal for communication satellites.

11.
a) Kepler found that r^3 / T^2 has a constant value for all the planets of the Solar System.

b) $F_c = F_G$ or $\frac{mv^2}{r} = \frac{GMm}{r^2}$

Simplifying gives: $v^2 = \frac{GM}{r}$ but $v = \frac{2\pi r}{T}$

So, $\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$

re-arranging: $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$ = constant

12.
a) GPE is the work done to move an object from infinity to a point within the gravitational field.
b) GPE must always be a negative quantity.